

FMA Team Mathematics Competition - Zone
YEAR 13 – 2015

Y13/1 If $A = \begin{bmatrix} -3 & 1 \\ -2 & 4 \\ 5 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & -3 \\ 0 & -2 \\ -2 & 4 \end{bmatrix}$ then what is $3A - 2B$?

Y13/2 What is the number of points (x, y) at which the parabola $y = x^2$ intersects the graph of the function $y = 1/(1 + x^2)$?

Y13/3 If the larger base of an isosceles trapezoid equals a diagonal and the smaller base equals an altitude, what is the ratio of the smaller base to the larger base?

Y13/4 what is the least positive number n such that $1 + 2 + \cdots + n > 100$?

Y13/5 All six digits of three 2-digit numbers are different. What is the largest possible sum of three such numbers?

Y13/6 What is the maximum value of the function $f(x) = \frac{\sin^3 x \cos x}{\tan^2 x + 1}$?

Y13/7 The surface area of a right rectangular prism (a box) is 48 square feet, and the sum of its length, width, and height is 13 feet. What is the length of the longest diagonal connecting two corners of the box?

Y13/8 The sale price of a shirt is 40% off its original price of \$100. An employee gets an additional 20% off this sale price. What would an employee pay for this shirt?

Y13/9 Six friends will exchange books in their book club. Each friend has one book to give to a friend, and will receive one book from a different friend. (No two friends trade books with each other.) In how many ways can the books be exchanged?

Y13/10 A bug is flying on a three-dimensional grid and wants to go from $(0,0,0)$ to $(2,2,2)$. It flies a distance of 1 unit at each step, parallel to one of the coordinate axes. How many paths can the bug choose which take only six steps?

Y13/11 Dean runs up a mountain road at 8 km per hour. It takes him one hour to get to the top. He runs down the same road at 12 km per

Y13/12 In a class of 100 students, there are 50 who play soccer, 45 who play basketball, and 50 who play volleyball. Only 15 of these students play all three sports. Everyone plays at least one of these sports. How many of the students play exactly two of these sports?

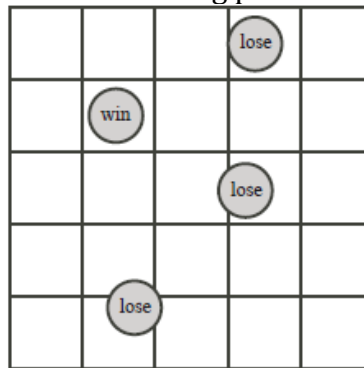
Y13/13 At a party, each person shakes hands with 5 other people. There are a total of 60 handshakes. How many people are at the party?

Y13/14 It rained on exactly 7 of the days during Jane's summer holiday trip. On each day that it rained, it rained either in the morning or the afternoon but not both. There were exactly 5 afternoons when it did not rain and exactly 6 mornings when it did not rain. How many days did the trip last?

Y13/15 What is the remainder when $x^{51} + 51$ is divided by $x + 1$?

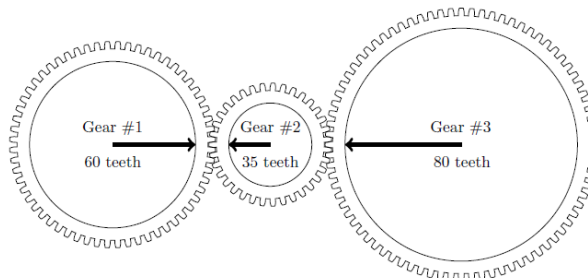
- Y13/16 A bag contains hundreds of glass marbles, each one coloured either red, orange, green or blue. There are more than 2 marbles of each colour. Marbles are drawn randomly from the bag, one at a time, and not replaced. How many marbles must be drawn from the bag in order to ensure at least three marbles of the same colour are drawn?

- Y13/17 A coin that is 8 cm in diameter is tossed onto a 5 by 5 grid of squares each having side length 10 cm. A coin is in a winning position if no part of it touches or crosses a grid line, otherwise it is in a losing position. Given that the coin lands in a random position so that no part of it is off the grid, what is the probability that it is in a winning position?

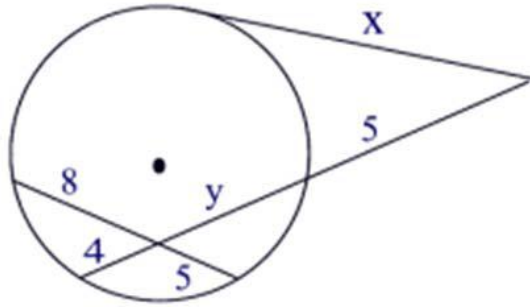


- Y13/18 Let $f(x)$ be a function such that $f(x) + f\left(\frac{1}{1-x}\right) = x$ for all x not equal to 0 or 1. What is the value of $f(2)$?

- Y13/19 How many rotations of Gear #1 are required before all three gears return to the position shown, with the arrows lined up again and pointing in the same directions as before? Gear #1 has 60 teeth, Gear #2 has 35 teeth and Gear #3 has 80 teeth



Y13/20 Given circle with tangent, secant and chord, Find the value of x .



TIE BREAKER

Y13/21 A solid cube is divided into two pieces by a single rectangular cut. As a result, the total surface area increases by a fraction f of the surface area of the original cube. What is the greatest possible value of f ?

Y13/22 You are told that 30 pupils have 25 different birthdays between them. What is the largest number of these pupils who could share the same birthday?
